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DETERMINATION MEASURE OF EFFICIENCY USING UNDESIRABLE OUTPUTS OF DEA

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In this paper, the possibility of suitable production is presented, and then a new method is suggested taking into account the existence of some undesirable components in the outputs and inputs of DMUs in the set.

1. Introduction

Data envelopment analysis (DEA) uses linear programming problems to evaluate the relative efficiencies and inefficiencies of peer decision making units (DMUs) which produce multiple outputs and multiple inputs. Once DEA identifies the efficient frontier, DEA improves the performance of inefficient DMUs by either increasing the inputs that are not allowed or only outputs that are allowed to increase. If one treats the undesirable outputs as inputs, the resulting DEA model does not reflect the true production process. Fare et al.[1] developed a non-linear DEA program to model the paper production system where the desirable outputs are increased and the undesirable outputs are decreased. Situations when some inputs need to be increased to improve the performance are also likely to occur. For example, in order to improve the performance of a waste treatment process, the amount of waste (undesirable input) to be treated should be increased rather than decreased as assumed in the standard DEA model. The current paper develops an alternative approach to treat both desirable and undesirable factors differently in the standard linear BCC (DEA) model of Banker et al. [2]. This preserves the linearity and convexity in the BCC model. The key to our approach is the use of DEA classification invariance under which classifications of efficiencies and inefficiencies are invariant to the data transformation.

This paper is structured as follows: section 2 gives definitions of proportionate PPS to undesirable inputs and outputs. The method for measuring efficiency with undesirable inputs and outputs is shown in section 3. An example with undesirable outputs, and then the conclusion will be given in Section 4 and finally, some concluding remarks will follow in section 5.

2. Production Possibility Set

Suppose we have n observations on n DMUs with input and output vectors

(x_j, y_j) for j = 1, 2,..., n. Let $x_j = (x_{m1},...,x_{mj})^T$ and $y_j = (y_{1j},...,y_{sj})$. All $x_j \in R^m$ and $y_j \in R^s$ and $x_j > 0$, $y_j > 0$ *for j = 1, 2,... n*. The input matrix *X* and output matrix *Y* can be represented as $X = [x_1,...,x_j,...,x_n]$, $Y = [y_1,...,y_j,...,y_n]$. Where *X* is a $(m \times n)$ matrix and *Y* is a $(s \times n)$ matrix.

The production possibility set T is generally defined as :

$$T = \{(x,y) \mid x \text{ can produce } y\} \quad (1)$$

In DEA, the production possibility set under a Variable Return to Scale (VRS) technology is constructed from the observed data (x_j, y_j) *for j = 1, 2, ..., n* as follows:

$$T = \left\{(x,y) \middle| x \geq \sum_{j=1}^{n} \lambda_j x_j,\ y \leq \sum_{j=1}^{n} \lambda_j y_j,\ \lambda_j \geq 0,\ \sum_{j=1}^{n} \lambda_j = 1, j = 1,...,n\right\}. \quad (2)$$

In the absence of undesirable factors when DMU_O, $o \in \{1,2,...,n\}$, is under evaluation, we can use the following BCC model:

$$\begin{aligned} \min\ & \theta \\ s.t\ & \theta x_o - X\lambda \geq 0 \\ & Y\lambda \geq y_o, \\ & 1^T \lambda = 1, \\ & \lambda \geq 0. \end{aligned} \quad (3)$$

Corresponding to each output *y, L(y)* is defined as the following:

$$L(y_j) = \{x | (x, y_j) \in T\}. \quad (4)$$

In fact, $L(y_j)$ is a function that y_j portrays to a subset of inputs so that inputs can produce y_j.

Now, suppose that some inputs are undesirable so input matrix *X* can be represented as $X=(X^d,X^u)^T$, where $X^d = (x^d_{1j},...,x^d_{m_1 j}), j = 1,...,n$ and $X^u = (x^u_{1j},...,x^u_{m_2 j})$, $j = 1,...,n$ *are* $(m_1 \times n)$ *and* $(m_2 \times n)$ matrixes that represent desirable (good) and undesirable (bad) inputs, respectively. And similarly, suppose that some outputs are undesirable outputs. Matrix Y can be represented as $Y = (Y^g, Y^b)^T$, where $Y^g = (y^g_{1j},...,y^g_{s_1 j}), j = 1,...,n$ and $Y^b = (y^b_{1j},...,y^b_{s_2 j}), j = 1,...n$ are $(s_1 \times n)$ and $(s_2 \times n)$ matrixes that represent desirable (good) and undesirable (bad) inputs, respectively.

Definition 1: DMU of $(x^d_1, x^u_1, y^g_1, y^b_1)$ is dominated to DMU of $(x^d_2, x^u_2, y^g_2, y^b_2)$ if $x^d_1 \leq x^d_2, x^u_1 \geq x^u_2, y^g_1 \geq y^g_2$ and $y^b_1 \leq y^b_2$ the unequal is strict at least in a component. So that:

$$\begin{pmatrix} -x_1^d \\ x_1^u \\ y_1^g \\ -y_1^b \end{pmatrix} \geq \begin{pmatrix} -x_2^d \\ x_2^u \\ y_2^g \\ -y_2^b \end{pmatrix}$$

Definition 2: DMU_O is efficient if in T there is no DMU to be dominant over it.

We consider the properties of the Production Possibility Set as the following:

(1) T is convex.

(2) T is closed.

(3) The monotony property of desirable inputs and outputs. So that,

$$\forall u \in R_+^{m_1}, v \in R_+^{s_1}, (x^d, x^u, y^g, y^b) \in T \Rightarrow (x^d + u, x^u, y^g - v, y^b) \in T$$

This is not necessarily established for undesirable factors, because in this case, T has no efficient DMU.

Now, the Production Possibility Set T satisfying (1) through (3) can be defined by

$$T = \left\{ (x^d, x^u, y^b, y^g) \middle| x^d \geq \sum_{j=1}^{n} \lambda_j x_j^d, x^u = \sum_{j=1}^{n} \lambda_j x_j^u, y^g \leq \sum_{j=}^{n} \lambda_j y_j^g, \right.$$
$$\left. y^b = \sum_{j=1}^{n} \lambda_j y_j^b, \quad \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j = 1, \ldots, n \right\}. \quad (5)$$

3. Undesirable Outputs in DEA

Suppose $DMU_o = (x_o^d, x_o^I, y_o^g, y_o^b)$ is unit under evaluation, corresponding to the input $x_o = (x_o^d, x_o^I)$ and using (2), $p(x_o^d, x_o^I)$ in defined as follows:

$$p(x_o^d, x_o^I) = \left\{ (y^g, y^b) \middle| (x_o^d, x_o^I, y^g, y^b) \in T \right\} \quad (6)$$

Now, we consider the subset of $p(x_o^d, x_o^I)$ as:

$$\partial^p p(x_o^d, x_o^I) = \left\{ (y^g, y^b) \middle| \forall (u,v) \geq 0, (u,v) \neq 0 \Rightarrow (y^g + u, y^b - v) \notin p(x_o^d, x_o^I) \right\} \quad (7)$$

That $\partial^p p(x_o^d, x_o^I)$ includes all inputs of the efficient DMUs which can produce (y_o^g, y_o^b).

For efficiency in output oriented; we must increase y_o^g and decrease y_o^b for moving our DMU to efficiency frontier of $\partial^p p(x_o^d, x_o^L)$ and hence we want to:

$$\overrightarrow{NE}^d (x_o, y_o) = \sup\{\beta \mid y_o + \beta d \in p(x_o)\}$$

Where $d = (d^g, d^b)$ is our direction to move our DMU to frontier and $d^g \in R_+^{S_1}, d \in R_-^{m_2}$ are increased desirable outputs and decreased undesirable outputs.

In this paper, we want to increase desirable outputs radial to efficiency frontier and hence: $d^g = y_o^g$ and decrease undesirable outputs radial to efficiency frontier, and hence $(d^I = -y_o^b)$.

The model to evaluate the efficiency of DMU$_0$ with the most decrease of y_o^g and the most increase of y_o^b is as follows:

$$
\begin{aligned}
&\beta_o^* = Max \qquad \beta_o \\
&s.t. \\
&\quad \sum_{j=1}^{n} \lambda_j x_j^d + s^- = x_o^d \\
&\quad \sum_{j=1}^{n} \lambda_J x_j^I = x_o^I \\
&\quad \sum_{j=1}^{n} \lambda_j y_j^g - s^+ = y_o^g + \beta_o d_o^g \\
&\quad \sum_{j=1}^{n} \lambda_j y_j^b = y_o^b + \beta_o d_o^b \\
&\quad \sum_{j=1}^{n} \lambda_j = 1 \\
&\quad \lambda_j \geq 0 \qquad for \quad all \quad j = 1,\ldots,n
\end{aligned} \tag{8}
$$

Theorem 1: The evaluated DMUo in model (8) is efficient if and only if:

1) $\beta_o^* = 1$
2) All slacks are zero for all optimal solutions.

Theorem 2: If (β^*, s^{+^*}) is optimal solution of model (8), then

$$(y_o^* + \beta_o^* d_o^g + s^{+^*}, y_o^b + \beta_o^* d_o^b) \in \partial^p p(x_o^d, x_o^I)$$

4. Numerical example

Consider five DMUs with one desirable input, one undesirable output and one desirable output.

Regarding Table 1 and Figure 1, it can be seen that DMUs A, B, and C are efficient and they are on the $\partial^s L(y_G^g)$. On the other hand, efficiency of other DMUs has been examined through their image on $\partial^s L(y_G^g)$. (Efficient Frontiers)

Similar discussion can be presented for the output oriented

DMUs	x^d	y^g	y^b	$1-\beta^*$
A	1	4	1	1
B	1	5	2	1
C	1	5	4	1
D	1	4	5	0.25
E	1	3	3	0.5

Table1. The inputs and outputs for 5 DMUs.

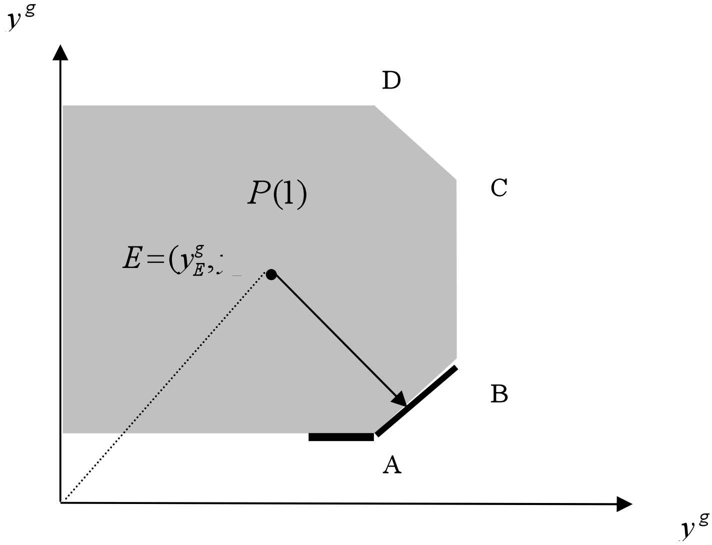

Figure 1. The graph of the L (y_G)

5. Conclusion

Throughout this paper, a new model is defined for the evaluation of efficiency where some inputs and outputs may be undesirable. Also, this model assures that the DMUs under evaluation will be compared with a corresponding unit of $\partial^s L(y^g_G)$.

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DEA-nın EFFEKTİV QİYMƏTİN ARZUOLUNMAZ ÇIXIŞLARLA MÜƏYYƏN EDİLMƏSİ

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XÜLASƏ

Məqalədə uyğun istehsalın mümkünlüyü göstərilir, sonra isə girişdə və çıxışda bəzi ölçü komponentlərin varlığı nəzərə alınmaqla yeni üsul təklif edilir.

ОПРЕДЕЛЕНИЕ ЭФФЕКТИВНОГО ЗНАЧЕНИЯ ДЕА С НЕЖЕЛАТЕЛНЫМИ ВЫХОДАМИ

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РЕЗЮМЕ

В статье показывается возможность соответствующий призводства, а потом предлагается новый метод с помощью компонентов некоторых мер при входе и выходе ДМИ.